A state-dependent hypoplastic model for methane hydrate-bearing sands

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Abstract

The mechanical behaviors of methane hydrate-bearing sands (MHBS) are largely affected by the presence of methane hydrate, temperature, and pore pressure. In this study, we present a simple hypoplastic model for MHBS. Methane hydrate saturation is included as a state parameter affecting the mechanical behaviors of MHBS. A new phase parameter is introduced to account for the coupled effects of temperature and pore pressure on the mechanical behaviors of MHBS. The phase parameter can be determined by a simple function of temperature and pore pressure. Comparison of the predictions with experiments shows that the model is able to capture the salient behaviors of MHBS.

Keywords: Hypoplastic constitutive model, Methane hydrate, Sands,
Phase state, Critical state

1. Introduction

Methane hydrate (MH) is a crystalline clathrate composed of methane and water molecules. It is widely distributed beneath permafrost and in marine continental margin sediments. MH can be used to extract methane gas, and has attracted global interests as a promising energy resource [1]. According to field investigations, MHs primarily exist in the forms of discrete nodules or veins in fine-grained sediments or disseminate in the pore spaces of coarse sands. Compared to fine-grained sediments, sand reservoirs are more

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attractive for MH exploitation due to the high-permeability of sand media and the high-concentration of MHs [2, 3]. However, the exploitation of MH may destabilize methane hydrate-bearing sands (MHBS) and potentially lead to the unsteady deformation of seabed [4], in-stability of production well [5], and even marine landslides [6]. It is therefore crucial to properly characterize the mechanical behaviors of MHBS for MH exploitation.

The mechanical behaviors of MHBS have been extensively investigated by laboratory tests. Sufficient experimental results [7–13] show that the stiffness, strength, and dilation of MHBS are enhanced by increasing MH saturation, which is defined as the ratio of MH volume to sediment pore volume. Besides, the mechanical behaviors of MHBS are markedly influenced by pore pressure and temperature [7, 13]. For example, the peak strength of MHBS increases by 18% when temperature decreases from 10 °C to 1 °C [7]; and a 40% increase of the peak strength is observed when pore pressure increases from 5 MPa to 10 MPa [13]. Hence, the MH saturation, temperature, and pore pressure dependences are important in modeling the mechanical behaviors of MHBS.

During the last decade, a great effort has been made in the development of constitutive models for predicting the mechanical behaviors of MHBS. Earlier models are usually developed based on the Duncan-Chang (DC) theory [14–16] and the Mohr-Coulomb (MC) theory [17–20]. In these constitutive models, MH saturation and temperature are considered as important factors that influence the mechanical behaviors of MHBS. The DC type models establish a stress-strain relationship through a hyperbolic function, in which the initial stiffness and the peak shear strength can be related to confining pressure [14], MH saturation [15], and temperature [14, 16]. With a nonlinear elastic formulation, the nonlinear behavior of MHBS can be described, while the applied hyperbolic function is usually too simple to fully capture the volumetric and strain softening behaviors. In the MC type models, the effect of MH saturation can be considered by including the dependency of cohesion on MH saturation [17–20]. However, without a yielding surface, the MC type models predict inaccurate volumetric deformation of MHBS under loading.

Although the aforementioned models are able to describe some basic features of MHBS, the prediction of the critical state behavior, an important aspect of the behavior of MHBS, is beyond the capacity of the models due to the limitation of model framework. In contrast to these models, the critical state models based on the Modified Cam-Clay framework consider the effects of MH saturation using the critical state parameters. The volumetric change,

strain softening, and MH-induced bonding can be reasonably described [21–25]. Thus, the critical state models are among the most common options in predicting the mechanical behavior of MHBS.

A closer inspection of the relevant literature reveals, however, that the effects of temperature and pore pressure, have rarely been included in available critical state models. Although an attempt has been made to account for the effects of temperature and pore pressure within the framework of bounding surface plasticity [23], the formulation of the constitutive equation is complicated, which may hinder the understanding and application of the model. From the practical point of view, a constitutive model with simple formulation while is capable of capturing the salient behaviors of MHBS is to be favoured.

Hypoplastic models describe the behaviors of granular materials by non-linear tensorial functions [26]. Unlike elastoplastic models, hypoplastic models need not to predefine yield surface, plastic potential, hardening rule, flow rule, and the decomposition of deformation into elastic and plastic parts [26]. Owing to their simple formulations with few model parameters, hypoplastic models have been widely used to model the behaviors of sands [27–30], clay [31, 32], frozen soil [33], and debris materials [34], etc. Recently, the hypoplastic model has also been applied to model the mechanical behaviors of MHBS [35]. The effects of MH saturation and confining pressure on the stress-strain and volumetric responses can be well captured. However, the parameters for different MH saturations and confining pressures are not unified, i.e., need to be recalibrated. Moreover, some salient factors, such as void ratio, temperature, and pore pressure, are not discussed.

This study aims to develop a hypoplastic model accounting for the effects of MH saturation, stress level, void ratio, temperature, and pore pressure. First, a simple hypoplastic model for sand [36] is enhanced by incorporating the critical state concept. Second, the effect of MH is incorporated by considering both the hydrate-induced bonding and the critical state line (CSL) being dependent on MH saturation, temperature, and pore pressure. In particular, a phase parameter is introduced to account for the coupled effects of temperature and pore pressure on the mechanical behaviors of MHBS. After that, the calibration of the model is illustrated. Last, the model is validated by comparing simulation results with experiments. The performance of the model and some limitations are also discussed.

2. Hypoplastic model with critical state for sand

Wu et al. [36] proposed a basic constitutive model for sand. This model contains tensor polynomial with three linear terms and one nonlinear term in strain rate:

$$\overset{\circ}{\boldsymbol{\sigma}} = C_1(\text{tr}\boldsymbol{\sigma})\dot{\boldsymbol{\epsilon}} + C_2(\text{tr}\boldsymbol{\sigma})(\text{tr}\dot{\boldsymbol{\epsilon}})\mathbf{I} + C_3\frac{\text{tr}(\boldsymbol{\sigma}\dot{\boldsymbol{\epsilon}})}{\text{tr}\boldsymbol{\sigma}}\boldsymbol{\sigma} + C_4(\boldsymbol{\sigma} + \boldsymbol{\sigma}^*)\|\dot{\boldsymbol{\epsilon}}\| \quad (1)$$

where C_i , ($i = 1, 2, 3, 4$) are dimensionless parameters, the sign tr denotes the trace of a tensor, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\dot{\boldsymbol{\epsilon}}$ is the strain rate (stretching) tensor, $\mathbf{I}$ is the second-order unit tensor, $\boldsymbol{\sigma}^*$ is the deviatoric stress tensor defined as $\boldsymbol{\sigma}^* = \boldsymbol{\sigma} - (\text{tr}\boldsymbol{\sigma})\mathbf{I}/3$, $\|\dot{\boldsymbol{\epsilon}}\|$ is the Euclidean norm of the strain rate, and $\overset{\circ}{\boldsymbol{\sigma}}$ is the Jaumann stress rate defined as:

$$\overset{\circ}{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\dot{\boldsymbol{\omega}} + \dot{\boldsymbol{\omega}}\boldsymbol{\sigma} \quad (2)$$

where $\dot{\boldsymbol{\sigma}}$ is the material time derivative of the Cauchy stress, and $\dot{\boldsymbol{\omega}}$ is the spin tensor.

This basic model takes the stress tensor as the only state variable. As a consequence, different parameters are required for the same material but with different initial densities. This shortcoming, however, can be readily remedied by integrating the critical state concept. To this end, a density function f_d [37] is introduced to account for the effect of void ratio. The basic model (1) is then recast as follows:

$$\overset{\circ}{\boldsymbol{\sigma}} = C_1(\text{tr}\boldsymbol{\sigma})\dot{\boldsymbol{\epsilon}} + (1 - f_d)C_2(\text{tr}\boldsymbol{\sigma})(\text{tr}\dot{\boldsymbol{\epsilon}})\mathbf{I} + C_3\frac{\text{tr}(\boldsymbol{\sigma}\dot{\boldsymbol{\epsilon}})}{\text{tr}\boldsymbol{\sigma}}\boldsymbol{\sigma} + f_d C_4(\boldsymbol{\sigma} + \boldsymbol{\sigma}^*)\|\dot{\boldsymbol{\epsilon}}\| \quad (3)$$

With the density function f_d , this model is able to consider variation of the density during shearing. Several forms of density functions can be found in the literature. In this work, a simple form suggested by Wang et al. [38] is used. It reads:

$$f_d = \left(\frac{e}{e_{\text{crt}}}\right)^\alpha \quad (4)$$

where e is the current void ratio, e_{crt} is the critical state void ratio, and α is a constitutive constant that controls the degree of strain softening [29, 38]. The value of f_d is less than 1 for a dense state, greater than 1 for a loose state, and equals 1 at the critical state. The critical state void ratio is determined

119 by:

$$e_{\text{crt}} = e_{\Gamma} - \lambda \ln\left(\frac{p'}{p_a}\right) \quad (5)$$

120 where λ is a parametric constants, $p' = \text{tr}\boldsymbol{\sigma}/3$ is the effective mean stress, p_a
 121 = 101.325 kPa is the atmospheric pressure, and e_{Γ} is the critical state void
 122 at the atmospheric pressure.

123 The constitutive equation (3), however, does not account for the influence
 124 of the initial void ratio on the initial stiffness, and usually gives rise to a very
 125 stiff stress-strain curve in triaxial test simulations. To modulate the shape
 126 of the stress-strain curve, the right side of Eq. (3) is multiplied by a stiffness
 127 function f_s proposed by Wu and Kolymbas [36]:

$$f_s = \frac{\exp[\beta(e_{\text{crt}} - e)]}{(1 + r)^2} \quad (6)$$

128 where β is a material parameter and r is the stress ratio $\|\boldsymbol{\sigma}^*\|/(\text{tr}\boldsymbol{\sigma})$. In this
 129 way, the constitutive model for the host sand is formulated as:

$$\dot{\boldsymbol{\sigma}} = f_s [C_1(\text{tr}\boldsymbol{\sigma})\dot{\boldsymbol{\epsilon}} + (1 - f_d)C_2(\text{tr}\boldsymbol{\sigma})(\text{tr}\dot{\boldsymbol{\epsilon}})\mathbf{I} + C_3 \frac{\text{tr}(\boldsymbol{\sigma}\dot{\boldsymbol{\epsilon}})}{\text{tr}\boldsymbol{\sigma}}\boldsymbol{\sigma} + f_d C_4(\boldsymbol{\sigma} + \boldsymbol{\sigma}^*)\|\dot{\boldsymbol{\epsilon}}\|] \quad (7)$$

130 3. Model extension to MHBS

131 The mechanical behaviors of MHBS have been widely investigated by tri-
 132 axial compression tests [7–13]. Experimental results reveal that the existence
 133 of MH enhances the strength and dilation of MHBS. A higher MH saturation
 134 usually leads to a larger cohesion and a higher location of the CSL on the
 135 $e - \log p'$ plane. In addition, the strength and dilatancy of MHBS can be
 136 enhanced by a lower temperature and a higher initial pore pressure.

137 To account for the aforementioned effects, we proceed to improve the
 138 sand model (7) in two aspects. First, an internal variable S is introduced
 139 to account for MH-induced cohesion. Second, a new phase parameter L' is
 140 proposed to describe the coupled effects of temperature and pore pressure
 141 on the cohesion and CSL of MHBS. The complete constitutive equations are
 142 given in ‘Appendix’ for clarity.

143 3.1. Incorporation of cohesion

144 Since the model (7) is homogeneous in stress, the limit surface in the
 145 principal stress space is a cone with its apex at the origin. As a result, Eq.

(7) allows no tensile stress. This shortcoming can be removed by transforming the stress tensor σ in Eq. (7) by an internal variable S :

$$\sigma_T = \sigma + S\mathbf{I} \quad (8)$$

where σ_T is a transformed stress tensor. In this way, the hydrate-induced cohesion can be included by replacing σ by σ_T in Eq. (7).

As indicated by experimental results [7, 8], MH saturation S_{MH} , temperature T , and pore pressure P have significant influences on the MH-induced cohesion. Therefore, the value of S should be as a function of S_{MH} , T , and P , i.e., $S = f(S_{MH}, T, P)$. This function will be formulated in subsection 3.3.3 after introducing the new phase parameter L' .

3.2. The effect of MH saturation

Experimental results reveal that the critical state line of MHBS will shift upward on the $e - \log p'$ plane with increasing MH saturation. Besides, the CSL of MHBS with a given S_{MH} is usually assumed to be parallel to that of the host sand [7], as shown in Fig. 1.

Figure 1: Critical state lines for MHBS [7].

Based on the above experimental observations, the effect of MH saturation on the CSL is suggested as:

$$e_{crt} = e_{\Gamma(MH)} - \lambda \ln(p'/p_a) \quad (9)$$

$$e_{\Gamma(MH)} = e_{\Gamma} + \xi S_{MH}^{x_1} \quad (10)$$

where ξ and χ_1 are model parameters. The parameter e_Γ is the same as that used in Eq. (5).

3.3. The effects of temperature and pore pressure

Previous experimental works show that the changes of temperature and pore pressure will lead to the variations of: (1) MH-induced cohesion and (2) the location of the CSL of MHBS [7, 8]. In the following, we proceed to consider the effects of temperature and pore pressure in a unified way by using a new phase parameter.

3.3.1. A new phase parameter L'

The stability of MHs with respect to temperature and pore pressure can be described by a phase parameter L defined by Hyodo et al. [39]. As illustrated in Fig. 2a, the phase parameter L is defined as the minimum distance from a phase state (Ω) to the stability boundary. This boundary differentiates the area where MHs are stable (solid) or unstable (gaseous). Generally, MHs are stable only when the phase state lies above the stability boundary. Within this area, a higher value of L indicates that the MH is more stable, and a higher strength can be gained [39], see Fig. 2b. Since the mechanical behaviors of MHBS are predominately governed by the properties of MH, the effects of temperature and pore pressure on the mechanical behaviors of MHBS can be considered using the phase parameter L [23, 40–43]. However, the value of L can be obtained only by iterative method in previous studies due to the high nonlinearity of the stability boundary. To facilitate the use of the phase parameter concept, we will propose a simple stability boundary based on experimental results [44–50].

As shown in Fig. 3a, the stability boundary can be represented by two lines in the $T_0/T - \ln(P/P_0)$ plane. This characteristic suggests that the new phase parameter for a given phase state (Ω) can be expressed as a piecewise function as follows:

$$L' = \begin{cases} \frac{|35 \times (T_0/T) + \ln(P/P_0) - 35|}{\sqrt{35^2 + 1^2}} & \Omega \in \text{area ABD;} \\ \frac{|7.5 \times (T_0/T) + \ln(P/P_0) - 7.5|}{\sqrt{7.5^2 + 1^2}} & \Omega \in \text{area CBD;} \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

Figure 2: (a) Normalized temperature and pressure stability boundary and (b) relationship between the strength of MHBS and L [39].

where P and T denote the current pore pressure and temperature, respectively; $P_0 = 2.23$ MPa and $T_0 = 273$ K are the reference pore pressure and temperature, respectively; area ABD and CBD are the acute angle areas bounded by ABD and CBD, respectively (see Fig. 3b, in which line BD is the angle bisector of ABC). Compared to the phase parameter given in previous literature [23, 40–43], the new phase parameter L' is much simpler and can be explicitly determined.

Figure 3: Stability boundary of methane hydrate in pure water: (a) experimental data and (b) the determining of the new phase parameter L' .

197 *3.3.2. The effect of L' on the CSL*

198 The relationship between the CSL of MHBS and the phase parameter L'
 199 can be established by assuming that the phase parameter changes only the
 200 position of the CSL rather than its slope (Fig. 4). The parameter controlling
 201 the position of the CSL of MHBS is considered to be dependent on the phase
 202 parameter L' . The CSL of MHBS thus has the following formulation:

$$e_{\text{crt}} = e_{\Gamma(L')} - \lambda \ln(p'/p_a) \quad (12)$$

$$e_{\Gamma(L')} = e_{\Gamma} + \xi L'^{\chi_2} \quad (13)$$

203 where χ_2 is an additional parameter controlling the effect of L' on the CSL
 204 of MHBS.

Figure 4: The influence of phase parameter L' on the CSL of MHBS (data from Hyodo et al. [7])

205 *3.3.3. The effects of S_{MH} and L' on the CSL and cohesion*

206 Based on Eqs. (10) and (13), we proposed the following functions to
 207 account for the effects of MH saturation and phase state on the CSL of
 208 MHBS:

$$e_{\text{crt}} = e_{\Gamma(MH, L')} - \lambda \ln(p'/p_a) \quad (14)$$

$$e_{\Gamma(MH, L')} = e_{\Gamma} + \xi S_{MH}^{\chi_1} L'^{\chi_2} \quad (15)$$

209 In addition, the effects of MH saturation and phase state on the MH-
 210 induced cohesion is suggested as:

$$S = aS_{\text{MH}}^b L' \quad (16)$$

211 where a and b are additional parameters controlling the effects of the new
 212 phase parameter and MH saturation on the MH-induced cohesion.

213 4. Calibration of the model

214 The proposed model for MHBS contains 14 parameters, 8 of which de-
 215 scribe the mechanical behaviors of the host sand and the remaining 6 account
 216 for the effects of MH saturation and the phase state. The value of the phase
 217 state parameter L' can be explicitly obtained by Eq. (11) for desired tem-
 218 perature and pore pressure. For instance, Table 1 shows the values of L' for
 219 desired temperature and pore pressure. Other parameters are determined in
 220 the following subsections.

Table 1: The values of L' for desired temperature and pore pressure

Temperature T [K]	Pore pressure P [MPa]	State parameter L'
274	10	0.0392
274	15	0.0508
278	5	0.0051
278	10	0.0249
283	10	0.0075

221 4.1. Calibration of parameters for the host sand

222 The parameters C_i , ($i=1, 2, 3, 4$) can be determined with a single triaxial
 223 compression test according to the procedure described by Wu and Bauer
 224 [27]. In this study, the material parameters determining C_i , ($i=1, 2, 3,$
 225 4) are $E_i/\sigma_c = 180$, $\nu_i = 0.2$, $\psi_{\text{crt}} = 0^\circ$, and $\phi_{\text{crt}} = 31^\circ$ [7], where E_i is
 226 the initial tangent modulus; $\sigma_c = 1$ MPa is the confining pressure in the
 227 triaxial compression test; ν_i is the initial Poisson ratio; ψ_{crt} is the critical
 228 state dilatancy angle; and ϕ_{crt} is the critical state friction angle. The critical
 229 state parameters e_Γ and λ can be determined by fitting the critical state line
 230 of the host sand [7], as shown in Fig. 4.

231 The parameters α and β can be calibrated by fitting the peak strength
 232 and modulus of MHBS (in the same single triaxial compression test). The
 233 parameters for modeling of the host sand [7] are shown in Table 2.

Table 2: Model parameters for the host sand

Parameter	Value
C_1	-50
C_2	-53.46
C_3	-436.48
C_4	-147.73
e_Γ	1.145
λ	0.126
α^*	0.8
β^*	3

* α and β should be re-calibrated for MHBS.

234 4.2. Calibration of parameters for MHBS

235 Apart from the 8 model parameters for the host sand, 6 additional pa-
 236 rameters are used to account for the effects of MH saturation, temperature,
 237 and pore pressure. As shown in Eqs. (15) and (16), both the MH saturation
 238 and the phase state influence the hydrate-induced cohesion and CSL on the
 239 $e - \ln p'$ plane. For the MH-induced cohesion, the internal variable S can be
 240 determined by back calculating the experimental data of the peak strength
 241 of MHBS [21]. As illustrated in Fig. 5, the internal variable S is taken as the
 242 mean effective stress p' corresponding to zero deviatoric stress q ($q = \sigma'_1 - \sigma'_3$,
 243 with σ'_1 and σ'_3 being the maximum and minimum principal stresses, respec-
 244 tively) in the peak strength envelope. Therefore, the parameters a and b can
 245 be obtained by fitting the variation of S against S_{MH} and L' .

246 As illustrated in Fig. 6, the value of $e_{\Gamma(\text{MH}, L')}$ in Eq. (15) is the intercept
 247 of the CSL on the y -axis. The parameters ξ , χ_1 , and χ_2 can be obtained by
 248 fitting the variation of $e_{\Gamma(\text{MH}, L')}$ against S_{MH} and L' . Note that the parame-
 249 ters α and β need to be re-calibrated for MHBS, which may be because the
 250 existence of MH alters the particle-scale property of sand grains. By fitting
 251 a single triaxial compression test on MHBS, the values of α and β are deter-
 252 mined as 0.3 and 5, respectively. The remaining 5 parameters are presented
 253 in Table 3.

Figure 5: Peak strength envelopes of MHBS at different hydrate saturations (data from Hyodo et al. [7]).

Table 3: The remaining parameters for MHBS

Parameter	a [MPa]	b	ξ	χ_1	χ_2
Value	359	5.815	6.526	2.486	0.523

254 5. Model performance

255 In this section, model predictions are compared with experimental results
 256 of MHBS. A series of numerical triaxial compression tests, considering dif-
 257 ferent confining pressures, initial void ratios, MH saturations, temperatures,
 258 and pore pressures, are carried out to validate this model. The parameters
 259 used in the simulations are summarized in Tables 2 and 3.

260 5.1. Model performance for the host sand

261 Figures 7 and 8 show the comparisons between the predictions and ex-
 262 periments on host sand with different confining pressures and void ratios,
 263 respectively. In experiments, the strength and stiffness of MHBS increase
 264 with the increase of the effective confining pressure, while the dilatancy is
 265 weakened by the increase of the void ratio. In addition, a strain-softening
 266 behavior is observed under a lower effective confining pressure, i.e., 1 MPa,
 267 while strain-hardening behaviors are observed under higher effective con-
 268 fining pressures of 3 and 5 MPa. Overall, the model can well predict the

Figure 6: The effects of S_{MH} and L' on the critical state line (data from Hyodo et al.[7]).

269 stress-strain and volumetric responses of the host sand for different confining
 270 pressures and void ratios observed in experiments [7].

Figure 7: Comparisons between predicted and experimental results for the host sand with different effective confining pressures: (a) stress-strain and (b) volumetric responses (data from Hyodo et al. [7]).

271 5.2. Model performance for MHBS

272 Figure 9 presents the comparisons between experimental and the pre-
 273 dicted results of MHBS with different MH saturations. Experimental results

Figure 8: Comparisons between predicted and experimental results for the host sand with different void ratios: (a) stress-strain and (b) volumetric responses (data from Hyodo et al. [7]).

274 indicate that, with the increase of S_{MH} , the MHBS samples show enhance-
 275 ment in strength, stiffness, and dilatancy. At an MH saturation of 53.1%, a
 276 significant dilative behavior is observed regardless of the adopted high effective
 277 confining pressure (5 MPa). Despite the slightly discrepancy, the model
 278 successfully captures the main effects of S_{MH} on the stress-strain behaviors
 279 and the volumetric responses of MHBS.

Figure 9: Comparisons between predicted and experimental results for MHBS with different MH saturations: (a) stress-strain and (b) volumetric responses (data from Hyodo et al. [7]).

280 Figures 10 and 11 illustrate the predicted mechanical behaviors of MHBS

281 with different confining pressures and void ratios, respectively. It can be ob-
 282 served in experiments that for MHBS with S_{MH} of 53.7%, the strength, stiff-
 283 ness, and dilatancy are enhanced by increasing effective confining pressures
 284 and decreasing void ratios. The proposed model can capture the changes
 285 of the stress-strain behaviors and volume change responses with varying
 286 effective confining pressures and void ratios. Admittedly, there are some
 287 discrepancies between the predicted stress-strain/volumetric responses and
 288 experimental data. The discrepancies maybe attributed to the effect of MH
 289 morphology, which is not considered in the current model. This effect will
 290 be discussed in Section 6.

Figure 10: Comparisons between predicted and experimental results for MHBS with different confining pressures: (a) stress-strain and (b) volumetric responses (data from Hyodo et al. [7]).

291 Figures 12 and 13 compare the measured and predicted results of MHBS
 292 under different temperatures and pore pressures. Experimental results show
 293 that when pore pressure increase from 5 MPa to 10 MPa, the stiffness of
 294 MHBS increase markedly; and the strength and dilatancy are also enhanced.
 295 In addition, when temperature decreases from 283 K to 274 K, obvious in-
 296 crease in the stiffness, strength, and dilatancy of MHBS are observed. The
 297 comparisons of experimental and numerical results show that the proposed
 298 model is able to sufficiently capture the above effects. Overall, the results
 299 predicted by our model are close to the experimental data.

Figure 11: Comparisons between predicted and experimental results for MHBS with different void ratios: (a) stress-strain and (b) volumetric responses (data from Hyodo et al. [7]).

Figure 12: Comparisons between predicted and experimental results for MHBS with different pore pressures: (a) stress-strain and (b) volumetric responses (data from Hyodo et al. [7]).

300 6. Discussion

301 The mechanical behaviors of MHBS show a strong dependency on MH
 302 saturation. With the increase of MH saturation, a significant change in me-
 303 chanical response will occur. Besides, the coupled effects of temperature
 304 and pore pressure are also important aspects of the mechanical behaviors of
 305 MHBS. So far, most of available constitutive models for MHBS take only the
 306 effect of MH saturation into account. One exception is the contribution made

Figure 13: Comparisons between predicted and experimental results for MHBS with different temperatures: (a) stress-strain and (b) volumetric responses (data from Hyodo et al. [7]).

by Ng et al. [23], who consider the coupled effects of temperature and pore pressure by a phase parameter similar to that in this study. However, the phase parameter in [23] is determined in an indirect way (e.g., by iterative method), while the phase parameter proposed in this study can be determined straightforward by a simple function. In addition, the formulation of the proposed model is much more simple than that of most available constitutive models for MHBS. Therefore, the proposed hypoplastic model possess great advantages in modeling the salient mechanical behaviors of MHBS. Besides, the proposed phase parameter may provide convenience in future modeling of the coupled effects of temperature and pore pressure.

Apart from MH saturation, temperature, and pore pressure, the MH morphology is also an important factor in controlling the mechanical behaviors of MHBS. The MH morphology is usually categorized into four types: pore-filling, load-bearing, cementation, and grain-coating [12]. In the past, the MH morphology is viewed to be controlled by hydrate formation environment. For example, in a free water environment, hydrate tends to form from dissolved gas molecules or gas bubbles, resulting in the pore-filling type or the load-bearing type; in a free gas environment, hydrate tends to form as the cementation type. Based on this idea, experiments performed in different hydrate formation environments (free gas/free water environment) [7, 8] were usually used for the validation of constitutive models in terms of different MH morphologies (cementation/pore-filling type) [23, 24, 51]. However, MH

329 morphology cannot be identified simply by the hydrate formation environ-
 330 nment. Based on high-quality tomography (e.g., micro-computed tomography
 331 and X-ray tomography), it is reported that pore-filling type MH is formed
 332 even in the free gas environment [52]. Moreover, reliable evidences that the
 333 transformation of MH morphology and coexistence of different MH morpholo-
 334 gies are increasingly reported [53, 54]. Therefore, cautions should be taken
 335 when using experimental data in terms of MH morphology since idealized
 336 pore-filling or cementation type MH may not physically exist.

337 The effect of MH morphology is not accounted for in this study because
 338 available experiments are not sufficient to isolate the effects of pore-filling,
 339 cementation, grain-coating, and load-bearing types of MH. Inevitably, the
 340 ignorance of MH morphology hinders the accuracy in model prediction. In
 341 addition, the accuracy in model prediction is also influenced by the adopted
 342 critical state parameter. Since the critical state parameters are determined
 343 at the end of shearing, where the critical state may not be reached [7], the
 344 performance of our model may be affected. To establish a comprehensive
 345 constitutive model for MHBS, the effect of MH morphology and the critical
 346 state behaviors of MHBS need further study.

347 7. Conclusions

348 A state-dependent critical state model is developed within the frame-
 349 work of hypoplasticity. The model is able to account for hydrate saturation,
 350 temperature, and pore pressure. A new phase parameter with explicit for-
 351 mulation is proposed, which includes the coupled effects of temperature and
 352 pore pressure on the mechanical behaviors of MHBS in a unified way. The
 353 model is validated with experimental results of MHBS obtained under a wide
 354 range of stress level, void ratios, hydrate saturation, temperature, and pore
 355 pressure. Comparisons of the predictions with experiments show the model
 356 is satisfactory in predicting the salient mechanical behaviors of MHBS.

357 Benefited from the non-linear tensorial functions, the hypoplastic model
 358 has the advantage of simplicity while retaining its ability to capture the
 359 salient behaviors of MHBS. These advantages make the hypoplastic model
 360 as an attractive alternative to the prevailing elastoplastic constitutive models
 361 for MHBS.

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368 Appendix

The complete hypoplastic equations for MHBS are illustrated as follows:

$$\dot{\sigma} = f_s \left[C_1(\text{tr}\sigma_T)\dot{\epsilon} + (1-f_d)C_2(\text{tr}\sigma_T)(\text{tr}\dot{\epsilon})\mathbf{I} + C_3 \frac{\text{tr}(\sigma_T \dot{\epsilon})}{\text{tr}\sigma_T} \sigma_T + f_d C_4 (\sigma_T + \sigma_T^*) \|\dot{\epsilon}\| \right]$$

where

$$\begin{cases} \sigma_T = \sigma + S\mathbf{I} \\ S = aS_{\text{MH}}^b L' \\ f_s = \frac{\exp[\beta(e_{\text{crt}} - e)]}{(1+r)^2} \\ f_d = \left(\frac{e}{e_{\text{crt}}}\right)^\alpha \\ e_{\text{crt}} = e_{\Gamma(\text{MH}, L')} - \lambda \ln(p'/p_a) \\ e_{\Gamma(\text{MH}, L')} = e_\Gamma + \xi S_{\text{MH}}^{\chi_1} L'^{\chi_2} \end{cases}$$

369 In the above equations, σ_T is the transformed stress tensor, $\dot{\sigma}$ is the
 370 Jaumann stress rate, σ is the Cauchy stress tensor, $\dot{\epsilon}$ is the stretching tensor,
 371 $\mathbf{I}$ is the second-order unit tensor, S is the internal variable e is the current void
 372 ratio, e_{crt} is the critical state void ratio, r is the stress ratio $\|\sigma^*\|/(\text{tr}\sigma)$, L' is
 373 the phase state parameter, $e_{\Gamma(\text{MH}, L')}$ is the critical state void of MHBS at the
 374 atmospheric pressure, e_Γ is the critical state void of sand at the atmospheric
 375 pressure p_a , and p' is the effective mean stress. The parameters of the model
 376 are C_1 , C_2 , C_3 , C_4 , e_Γ , λ , α , β , a , b , ξ , χ_1 , χ_2 , and L' .

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