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Extension of a basic hypoplastic model for overconsolidated clays

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Abstract

This paper presents a hypoplastic constitutive model for overconsolidated clays. The model is developed based on a basic model proposed recently for sand. New density and stiffness factors are introduced to account for history dependence. The model is capable of predicting a desirable asymptotic state boundary surface for overconsolidated clays. Validation with experimental results shows this model can properly describe the main features of overconsolidated clays.

Keywords: Hypoplastic model, Critical state, Overconsolidation, Response envelope, Asymptotic state boundary surface

1. Introduction

Constitutive modeling of overconsolidated (OC) clays has a long history, pioneered by the Modified Cam-clay (MCC) model based on the critical state concept [32, 33]. The MCC model is capable of describing the behavior of normally consolidated and lightly OC clays. The capacity of the model for OC clays has been largely enhanced by introducing various concepts, such as bounding surface [6] and subloading surface [12]. These theories can be integrated with various dilatancy relations to form constitutive models for overconsolidated soils [7, 47, 48, 49]. An inspection of the relevant literature reveals, however, that most of the existing models for OC clays are based on elastoplasticity.

Hypoplastic models are developed as an alternative approach to the prevailing elastoplastic models in describing the overconsolidation behavior. Following the pioneer work by Kolymbas and Wu [17, 18, 40, 41], different versions of hypoplastic models were developed for sand [4, 9, 37]. These models served as the basis for the model development of clay. The first attempt to extend hypoplastic model for clay was made by Niemunis [26, 27], who combined the critical state concept with hypoplasticity to describe the behavior of OC clays. Meanwhile, Herle and Kolymbas [14] modified the model by von Wolfferdorff [37] (VW model) for soils with low friction angle. Based on these works, Mašín [21] proposed the first hypoplastic model for clay (DM model), which employs a density factor to describe the consolidation history.

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However, later Mařín himself found that the DM model gives rise to an unrealistic asymptotic state boundary surface (ASBS) when a certain combination of parameters is used [23]. The reason for this deficiency is that the adopted density factor is not only dependent on the consolidation history but also related to other material parameters. To address this issue, Mařín [22, 23] proposed a new form of hypoplastic models with explicit ASBS. Nevertheless, the original form is still valuable for further developments and enhancements. For instance, Wu et al. [45] reported a basic hypoplastic model for sand. This model employs the basic form of previous hypoplastic models but additionally includes a new tensorial term, which provides an opportunity to refine the modelling of the dilatant behaviour and asymptotic state.

In this work, we will extend the basic model [45] for OC clays. In the following, we first inspect the limitation of the basic model. Next, a detail process for developing the model for OC clays is presented. A new density factor is introduced to account for consolidation history. The employment of the density factor allows the model to predict reasonable ASBSs for OC clays. The characteristics of the model are then investigated by a parametric study. At the end, model performance is demonstrated by simulating several element tests, and comparing the prediction with experiments on different OC clays.

2. Theory of hypoplastic constitutive model

2.1. General framework

To gain perspective, we start with the rate-form constitutive equation by Wu and Kolymbas [41], which has the following general form:

$$\dot{\mathbf{T}} = \mathcal{L} : \mathbf{D} + \mathbf{N} \|\mathbf{D}\| \quad (1)$$

where $\mathcal{L}$ and $\mathbf{N}$ are fourth- and second-order tensors, respectively. $\mathbf{D}$ is the strain rate (stretching) tensor. $\|\mathbf{D}\| = \sqrt{\text{tr}(\mathbf{D}^2)}$ stands for the norm of the strain rate tensor. $\dot{\mathbf{T}}$ is the Jaumann stress rate defined by:

$$\dot{\mathbf{T}} = \overset{\circ}{\mathbf{T}} + \mathbf{T}\mathbf{W} - \mathbf{W}\mathbf{T} \quad (2)$$

where $\overset{\circ}{\mathbf{T}}$ is the material time-derivative of the Cauchy stress tensor and $\mathbf{W}$ the spin tensor.

Bear in mind that the failure criterion and flow rule of the model (1) is the interactional outcome of $\mathcal{L}$ and $\mathbf{N}$. This implies the intensity and direction of flow are mutually dependent. To remedy this shortcoming, Niemunis [26] proposed a simple rearrangement of the constitutive framework (1) by introducing a tensor function $\mathbf{B}$:

$$\mathbf{B} = -Y \frac{\mathbf{m}}{\|\mathbf{m}\|}, \text{ and } \mathbf{N} = \mathcal{L} : \mathbf{B} \quad (3)$$

where the tensor $\mathbf{B}$ includes both the flow direction and intensity of the flow by the function Y and a tensor $\mathbf{m}$, respectively.

With the above rearrangement, a generalized form of hypoplastic models is obtained as follows:

$$\dot{\mathbf{T}} = f_s \mathcal{L} : \left(\mathbf{D} - f_d Y \frac{\mathbf{m}}{\|\mathbf{m}\|} \|\mathbf{D}\| \right) \quad (4)$$

where f_s and f_d are the stiffness and density factors, accounting for the effects of stress level and density, respectively [21, 26].

2.2. A basic hypoplastic model

To develop a new constitutive model for OC clays, we start by considering the following basic model proposed by Wu et al. [45]:

$$\dot{\mathbf{T}} = C_1(\text{tr}\mathbf{T})\mathbf{D} + C_2(\text{tr}\mathbf{D})\mathbf{T} + C_3 \frac{\text{tr}(\mathbf{T}\mathbf{D})}{\text{tr}\mathbf{T}} \mathbf{T} + C_4(\mathbf{T} + \mathbf{T}^*)\|\mathbf{D}\| \quad (5)$$

in which C_i ($i = 1, 2, 3, 4$) are material constants. $\mathbf{T}^*$ is the deviatoric stress tensor defined as $\mathbf{T}^* = \mathbf{T} - \mathbf{I}/3\text{tr}\mathbf{T}$ with $\mathbf{I}$ being the unit tensor. The material constants can be related to some well established parameters in soil mechanics, such as the initial tangential modulus E_i , the initial Poisson's ratio ν_i , the critical friction angle ϕ_c , and dilatancy angle ψ . Moreover, for critical state with $\psi = 0$, the model (5) can be further rearranged as the following form:

$$\dot{\mathbf{T}} = f_s \left[\mathbf{D} + c_2(\text{tr}\mathbf{D})\hat{\mathbf{T}} + a^2 \text{tr}(\hat{\mathbf{T}}\mathbf{D})\hat{\mathbf{T}} + a(\hat{\mathbf{T}} + \hat{\mathbf{T}}^*)\|\mathbf{D}\| \right] \quad (6)$$

where $\hat{\mathbf{T}}$ is the normalized stress defined as $\hat{\mathbf{T}} = \mathbf{T}/\text{tr}(\mathbf{T})$; the multiplier c_2 permits a refined modelling of the dilatancy behavior. The constant a accounts for the critical state through:

$$a = \frac{\sqrt{3}(3 - \sin\phi_c)}{2\sqrt{2}\sin\phi_c} \quad (7)$$

Note that the equation (6) is a special case of equation (5) with $\psi = 0$. This basic model has several merits such as simplicity and robustness in numerical simulation. These advantages make it an interesting stand-alone model for many applications[30, 31, 36, 38, 46] and further developments [19, 20, 39].

2.3. Limitation of the basic model for OC clays

The performance of the basic model is well documented in the literature [45]. The model is capable of modeling the basic behavior of normally consolidated clay, such as failure and contractive deformation. However, there exist some shortcomings for modeling OC clays. The most important ones are summarized as follows:

- The consolidation history is not incorporated in this model, thus it cannot describe strain softening and volume dilatancy of OC clays.
- This model does not include the swelling index κ^* . Therefore, the directional stiffness for loading and unloading is not adjustable.

- This model has a constant stiffness factor f_s . It is adequate to describe the initial shear modulus of normally consolidated soil but it will underestimate significantly the initial shear moduli of OC clays.

Note that the VW model and DM model have similar forms with model (6) except that the second term $\text{tr}(\mathbf{D})\hat{\mathbf{T}}$ is not included. These two models address some of the above mentioned shortcomings. However, the VW model is seriously flawed because the underlying basic equation does not perform properly [45]. The DM model also suffers from some shortcomings [23].

3. Proposed model for OC clays

To address the aforementioned shortcomings of the basic model (6), we proceed to extend it by considering some salient features of OC clays.

3.1. Model framework

We first consider the linear behavior of the model (6) by manipulating the linear terms. To do so, the model (6) can be expressed as follows:

$$\dot{\mathbf{T}} = c_1 \mathcal{L}_1 + c_2 \mathcal{L}_2 + c_3 \mathcal{L}_3 + \mathbf{N} \quad (8)$$

where $\mathcal{L}_i$ ($i = 1, 2, 3$) are the linear tensorial function in equation (6) with c_i ($i = 1, 2, 3$) being the material constants.

The effects of the linear terms can be best appreciated by considering the response envelope. For a rosette of strain rates with a constant magnitude, the response envelope at a given initial stress state $\mathbf{T}_0$ is the surface spanned by all stresses $\mathbf{T} = \mathbf{T}_0 + \Delta \mathbf{T}$ [10]. Figure 1 shows some response envelopes depicted on the Rendulic plane with $T_{22} = T_{33}$. the response envelope of the model (6) represents an ellipse. Obviously, the multiplier c_1 scales the magnitude of $\mathbf{D}$, and thus can expand or shrink the response envelopes. The multipliers c_2 and c_3 have similar effects, that is increase c_2 and c_3 gives rise to longer major axis while does not change the minor axis of the ellipse.

Note that the distance from the initial stress $\mathbf{T}_0$ to any point on the response envelope represents the stiffness along certain strain rate. As shown in Figure 1(a), the vectors $|\vec{o}\vec{a}|$ and $|\vec{o}\vec{b}|$ represent, respectively, the directional stiffness of loading and unloading at a isotropic stress state. $|\vec{o}\vec{c}|$ and $|\vec{o}\vec{d}|$ denote the directional stiffness with isochoric strain. For clays, the ratio $|\vec{o}\vec{a}|/|\vec{o}\vec{b}|$ is specified by the loading and swelling indexes, i.e., κ^* and λ^* . It is known that a desirable ratio of $|\vec{o}\vec{a}|$ to $|\vec{o}\vec{c}|$ can be achieved by implementing two scale multipliers with $\mathcal{L}_1$ and $\mathcal{L}_3$ [14]. Analogously, with the help of the term $c_2 \mathcal{L}_2$, an adjustable ratio of loading to unloading stiffness is available by introducing the swelling index κ^* .

Another remark concerns with the enhancement of the initial shear stiffness, G_i , for OC clays. This subject has been investigated by several experiments [1, 2]. The experimental results indicate that the dependence of G_i on stress level and OCR can be described by exponential functions [11]. However, hypoplastic models usually underestimate G_i of OC clays, which is an intrinsic deficiency of many hypoplastic models. As

Figure 1: The effects of the tensorial function $\mathcal{L}$ on response envelopes

illustrated in Figure 1, the initial shear stiffness can be enhanced by multiplying the whole model. Following this way, we introduce an additional multiplier into the model (4):

$$\dot{\mathbf{T}} = m_R f_s \mathcal{L} : \left(\mathbf{D} - f_d Y \frac{\mathbf{m}}{\|\mathbf{m}\|} \|\mathbf{D}\| \right) \quad (9)$$

where m_R represents the stiffness enhancement of OC clays. In the following, the elements for the model (9) are derived. The complete formulations are given in the appendix.

3.2. Tensor function $\mathcal{L}$

The linear tensorial function of model (6) is considered for the new model. It can be written as:

$$\mathcal{L} = c_1 \mathcal{I} + c_2 \hat{\mathbf{T}} \otimes \mathbf{1} + c_3 \hat{\mathbf{T}} \otimes \hat{\mathbf{T}} \quad (10)$$

where $\mathbf{I}$ and $\mathbf{1}$ are the fourth-order and the second-order unit tensor, respectively.

Next, we proceed to determine the constants c_1 , c_2 , c_3 by assuming that the proposed model has the same form with the model (6) at an isotropic stress state. Therefore, the model (9) can be expressed with the following decomposed forms:

$$\dot{p} = -\frac{1}{3}f_s \left[\left(c_1 + c_2 + \frac{1}{3}c_3 \right) D_v + c_3 \hat{q} D_q + a \sqrt{\frac{1}{3}D_v^2 + \frac{3}{2}D_q^2} \right] \quad (11a)$$

$$\dot{q} = f_s \left[\frac{3}{2}c_1 D_q + \left(c_2 D_v + \frac{1}{3}c_3 D_v + c_3 \hat{q} D_q \right) \hat{q} + 2\hat{q}a \sqrt{\frac{1}{3}D_v^2 + \frac{3}{2}D_q^2} \right] \quad (11b)$$

where $\dot{p}$ and $\dot{q}$ are the rates of the mean stress and the deviatoric stress, respectively. $\hat{q} = q/\text{tr}(\mathbf{T})$ is the normalized deviatoric stress. D_v and D_q are the volumetric strain and deviatoric strain, respectively.

For an isotropic stress state, we have initially $\hat{q} = 0$, $D_q = 0$, and $D_v < 0$, and thus equation (11a) becomes:

$$-\dot{p}_i = \frac{1}{3}f_s \left(c_1 + c_2 + \frac{1}{3}c_3 - \frac{\sqrt{3}}{3}a \right) D_v \triangleq K_i^+ D_v \quad (12)$$

where $K_i^+ = f_s(c_1 + c_2 + c_3/3 - a\sqrt{3}/3)/3$ represents the initial bulk modulus of the soil for loading.

Since the modification of the model does not influence the formulation of the model (6) at an isotropic stress state, and therefore we have:

$$c_3 = a^2 + 3(1 - c_1) \quad (13)$$

For unloading in an isotropic compression test, we have $D_v > 0$. The rate of mean stress can be gained as follows:

$$-\dot{p}_i = \frac{1}{3}f_s \left(c_1 + c_2 + \frac{1}{3}c_3 + \frac{\sqrt{3}}{3}a \right) D_v \triangleq K_i^- D_v \quad (14)$$

in which $K_i^- = f_s(c_1 + c_2 + c_3/3 + a\sqrt{3}/3)/3$ represents the initial bulk modulus of the soil for unloading.

On the other hand, in a drained test with constant volume change, i.e., $D_v = 0$ (isochoric shear), the equation (11b) is recast to:

$$\dot{q}_i = \dot{q}|_{\hat{q}=0} = \frac{3}{2}f_s c_1 D_q \triangleq 3G_i D_q \quad (15)$$

wherein $G_i = 0.5f_s c_1$ is the initial tangential shear modulus for isochoric shear.

Comparison between the initial tangential bulk modulus in loading and unloading conditions yields $K_i^+/K_i^- = \kappa^*/\lambda^*$. In addition, the ratio of K_i^+ to G_i can be considered as a material constant v_i . Combining the equations (12 to 15) and making use of the

above conditions, we obtain the formulations for the multipliers c_1, c_2 :

$$c_1 = \frac{2\sqrt{3}a(\gamma - 1)}{9v_i}, \quad c_2 = \frac{1}{3}(\sqrt{3}a\gamma - a^2 - 3) \quad (16)$$

with γ being the ratio $(\lambda^* + \kappa^*)/(\lambda^* - \kappa^*)$. The third constant c_3 is then obtained from equation (13).

3.3. Flow rule $\mathbf{m}$ and limit stress condition Y

The hypoplastic flow rule $\mathbf{m}$ indicates the flow direction, which should obey the conditions $\mathbf{m} \sim \hat{\mathbf{T}}^*$ at a critical state, and $\vec{\mathbf{m}} = \mathbf{I}/\sqrt{3}$ at an isotropic stress state [26]. The tensorial function $\mathbf{B}$ of the model (6) fulfils these requirements. In this paper, a similar form including Matsuoka-Nakai failure criterion is adopted:

$$\mathbf{m} = -\hat{\mathbf{T}}^* - \frac{F^2 - a^2 \|\hat{\mathbf{T}}^*\|^2}{F^2(1 + c_2) + a^2 \|\hat{\mathbf{T}}\|^2} \hat{\mathbf{T}} \quad (17)$$

where F is an interpolation function to account for the Matsuoka-Nakai failure criterion [21, 37].

The failure criterion is obtained by incorporating the Matsuoka-Nakai failure criterion into the non-linearity Y . In this work, a similar Y as that used in [21] is adopted. It reads:

$$Y = \frac{(\gamma - 1)(I_1 I_2 / I_3 - 9) + 8 \tan^2 \phi_c}{8 \gamma \tan^2 \phi_c} \quad (18)$$

3.4. The density factor f_d

The factor f_d is used to consider the consolidation history. Bearing in mind that $f_d = 1$ shall be guaranteed for normal consolidation ($\text{OCR} = 1$), we proceed to introduce the following simple expression for the density factor:

$$f_d = R^\alpha \quad (0 < R \leq 1) \quad (19)$$

where α is a model parameter; R is the stress ratio representing the overconsolidation [47]. A smaller value of R corresponds to a larger OCR to give:

$$R = \frac{p_e^+}{p_e^*} \quad (20)$$

in which the p_e^+ and p_e^* are the pre-consolidation pressure in MCC model and the Hvorslev equivalent pressure respectively. Their definitions are shown in Figure 2 with the formulations given as follows:

$$p_e^+ = p \left[1 + \left(\frac{q}{M(\theta)p} \right)^2 \right], \quad p_e^* = \exp \left[\frac{N - \ln(1 + e)}{\lambda^*} \right] \quad (21)$$

in which the inclination M corresponds to the stress ratio at the critical state. It is interpolated with the function F to define the critical state line at different Lode angles θ , i.e., $M = 6F\sin\phi_c/(3-\sin\phi_c)$; N stands for the logarithmic value of the specific volume at the reference stress $p_r = 1$ kPa; the index λ^* is the slope of the normal compression line in the double logarithmic $\ln(1+e) - \ln p$ plane.

Figure 2: The definition of (a) Hvorslev equivalent pressure p_e^* , and (b) the stress ratio R

3.5. The stiffness factor f_s

The factor f_s can be obtained from the pre-defined isotropic normal compression line. After Butterfield [5], the normal compression line is expressed as:

$$\ln(1+e) = N - \lambda^* \ln(p/p_r) \quad (22)$$

Time differentiation of equation (22) gives rise to the rate form:

$$\dot{p} = \frac{-p\dot{e}}{\lambda^*(1+e)} \quad (23)$$

Comparing equations (23) and (12), and making use of $D_v = \dot{e}/(1+e)$, the expression for f_s is obtained:

$$f_s = \frac{\sqrt{3}\text{tr}\mathbf{T}}{2a} \left(\frac{1}{\lambda^*} - \frac{1}{\kappa^*} \right) \quad (24)$$

It can be seen that the stiffness factor f_s includes both the strength parameter a and the deformation parameters (λ^*, κ^*) through a simple formulation.

3.6. The multiplier m_R

In this paper, we propose a simple formulation for m_R :

$$m_R = \exp[\beta(1 - R)] \quad (25)$$

where β is a constant controlling the influence of consolidation history on the initial shear modulus. Obviously, we have $m_R = 1$ for normal consolidation with $R = 1$; for extreme case with $R \rightarrow 0$, the multiplier $m_R = \exp(\beta)$. If needed, β can be considered as a material parameter. In the present simulation, however, a fixed value $\beta = 1$ is used.

4. Model performance

In this section, we discuss the basic properties of the proposed model. First, a parametric study is performed to facilitate the selection of proper parameters in numerical simulation. Next, the performance of the proposed model is presented. Additionally, the influence of parameters on the shape and size of the ASBS is investigated.

4.1. Model parameters

The proposed hypoplastic model contains six constitutive parameters, i.e., ϕ_c , N , λ^* , κ^* , v_i , and α . The first five have the same physical interpretation as those used in the MCC model: ϕ_c is the critical state friction angle; N is the value of $\ln(1+e)$ at the isotropic normal compression line for $p = 1$ kPa; λ^* is the slope of the isotropic normal compression line on the $\ln(1+e) - \ln p$ plane; κ^* denotes the slope of unloading line in the same plane, and v_i is ratio between the initial bulk and shear moduli. Since the first five parameters of our model have the same physical meaning as those in the DM model. In the following, therefore, a series of triaxial tests are simulated to show the effects of α . In the simulations, two different conditions with OCR = 5 and 1.2 are considered. The confining pressure is assumed to be 100 kPa. The parameters used in the simulations are listed in Table 1.

Table 1: Parameters for model performance

Parameter	$\phi_c/^\circ$	λ^*	κ^*	N	v_i	α
Model test	30	0.1	0.01	1.0	0.2	0.5, 1.0, 1.5

Figure 3 shows the stress – strain relationship and effective stress paths of the model. A close inspection of the results reveals that α mainly influences the peak friction angle

Figure 3: Influence of the parameter α on the stress-strain and stress path curves in drained and undrained triaxial compression and extension tests

of OC clays. As shown in Figure 3(a, c), The parameter α governs the rate to achieve the critical state; a smaller α leads to a faster convergence to the critical state. In addition, α influences the effective stress paths in triaxial tests. As shown in Figure 3(b, d), the normalized stress paths expand out with the increase of α in triaxial compression tests. However, this influence is less significant in triaxial extension tests. Note that in the DM model α is a variable related to λ^* , κ^* , and ϕ_c . In a recent model by Mařín [23], a fixed value $\alpha = 2$ is adopted for numerical simulations.

4.2. Influence of parameters on the ASBS

Asymptotic behaviors are among the most significant mechanical properties of clays. For instance, one of the most important asymptotic states of clay is the well - known critical state behavior, which describes the asymptotic behavior of clay with constant volume during shearing [22, 23, 24]. The asymptotic states of a hypoplastic model can be achieved after a sufficiently long proportional stretching, i.e. stretching with a constant direction of the strain rate. This leads to the ASBS that bounds all admissible asymptotic states in the stress - void ratio space.

The size of ASBS should be controlled by the critical state friction angle ϕ_c . As shown in Figure 4, the ASBS of the proposed model expands with the increase of ϕ_c ,

This expansion does not leads the ASBS to the tensile stress region. It is found that the ASBS of our model is close to the elliptic shape of the yield surface of the MCC model [32] in the range $p > p_e^*/2$. The elliptic shape, however, becomes narrow in the range $p < p_e^*/2$, coinciding with the nonlinear Hvorslev surface on the dry side. This property ensures the model predicts properly the peak strength of heavily OC clays [3, 34].

Figure 4: Asymptotic state boundary surface for $\phi_c = 15^\circ, 20^\circ, 25^\circ, 30^\circ, 35^\circ, 40^\circ$

Mašín and Herle [24] reported that the ratio λ^*/κ^* has a significant influence on the shape of the ASBS predicted by the DM model. As shown in Figure 5(a), for high ratios of λ^*/κ^* , the DM model predicts unrealistic shape of the ASBS. However, this shortcoming does not exist in the proposed model. As shown in Figure 5(b), the proposed model predicts very similar ASBS with varying ratios of λ^*/κ^* , even with high values of λ^*/κ^* the shape of ASBS is reasonable.

For the DM model, the dependence of ASBS on the ratio of λ^*/κ^* is due to the connection between λ^*/κ^* and the parameter α . As α is not an independent parameter in the DM model, high ratio of λ^*/κ^* will give rise to small value of α . In the proposed model, however, the parameter α is given as an independent material constant, and thus the ratio of λ^*/κ^* does not influence α . As shown in Figure 6, increase of α gives rise to expansion in the wet side ($p/p_e > 0.5$), and shrinkage in the dry side ($p/p_e < 0.5$). This is consistent with the results presented in Figure 3, that the increase of α leads to increased peak friction angle of OC clays, and this influence becomes more obvious with low value α . However, the shape of ASBS remains almost unchanged with α increasing from 1.0 to 3.0.

5. Validation with experimental data

In this section, model predictions are compared with experimental results on both remoulded and undisturbed OC clays. Different types of triaxial tests, including isotropically consolidated undrained triaxial tests (CIU), isotropically consolidated drained triaxial tests with constant mean pressure (CIDP) and constant confining pressure (CID), are simulated to validate this model. Parameters used in the simulations are summarised

(a)

(b)

Figure 5: Influence of ratios λ^*/κ^* on the shape of ASBS predicted by (a) Mašín's model [21] and (b) proposed model

Figure 6: The influence of α on the shape of the ASBS of the proposed model

in Table 2. The first five parameters are collected from the literature, while the parameters v_i and α are obtained through optimization methods [16, 50, 51] to gain the best

match with experimental results.

Table 2: Parameters for simulation of different clays

Soils	$\phi_c/^\circ$	λ^*	κ^*	N	v_i	α
Wenzhou clay [8]	29.8	0.0684	0.025	0.95	1.0	0.89
Fujinomor clay [25]	33.7	0.0445	0.0108	0.86	0.6	0.5
Weald clay [29]	23.5	0.059	0.028	0.8	1.5	1.0
Bangkok stiff clay 1 [13]	31.5	0.032	0.0072	0.89	1.2	0.5
Bangkok stiff clay 2 [13]	30.2	0.0505	0.02	0.892	0.6	2.0

5.1. CIU test, Remoulded Wenzhou clay

The Wenzhou clays are mainly marine deposits from Wenzhou, eastern coastal China. The deposit usually reaches a depth of 150 m and exhibits evident heavily OC behavior. A series of triaxial compression tests on remoulded clay was carried out by Gu et al. [8] using a GDS device. The samples contain clay fraction of 63.8% and a fine fraction of 93.4%. The natural water content can reach to $w_L = 67.7\%$ with plasticity index $I_p = 37.2$ and initial void ratio, e_0 , ranging from 1.55 - 1.59 [8].

Figure 7: Comparison between experimental and numerical undrained triaxial tests on OC Wenzhou clay. Experimental data from Gu et al. [8]

Four samples were initially subjected to pre-consolidation pressure (p_c) of 100, 200, 400, 800 kPa. After the completion of pre-consolidation, the effective confining pressure was reduced to $p_0 = 100$ kPa in order to produce OC samples with specific OCR values. The test results are presented in Figure 7 together with model prediction. As shown in Figure 7(a), the samples with different OCRs exhibit various shear strengths, this history dependence can be predicted by the proposed model. The peak strength can also be captured quite accurately. However, the stress – strain curves agree not very well at the bends of the curves, as it underestimates the initial stiffness and also the degradation rate of the stiffness. Figure 7(b) shows that the predicted effective stress paths agree well with experimental results for heavy consolidation, while less well prediction is observed for normal consolidation.

5.2. CIDP test, Remoulded Fujinomori clay

Experimental data on reconstituted Fujinomori clay by Nakai and Hinoko [25] is a classical set of experiments used in the literature to evaluate different constitutive models within the framework of elastoplasticity [47]. The test was carried out with constant mean effective stress. Both drained triaxial compression and extension tests were performed on OC Fujinomori clay with OCR from 1 to 8; The sample with OCR = 8 was tested at mean effective stress of $p_0 = 98$ kPa, while other samples were tested at mean effective stress of 196 kPa.

The predicted stress - strain and volumetric change curves using the proposed hypoplastic model are shown in Figure 8. Good agreement in the stress ratio curves is achieved in the compression tests, while the model underestimates the peak strength in the extension tests. A more significant difference is observed in the results of volumetric strain, especially in the extension tests. Nevertheless, the main features of the variation of stress ratio and volumetric strain can be reasonably captured by the proposed model.

5.3. CIU and CID test, Remoulded Weald clay

A series of tests have been carried out by Parry [29] on reconstituted Weald clay with both normal consolidated ($p_c = 206.8$ kPa, OCR = 1) and heavy consolidated ($p_c = 827.4$ kPa, OCR = 12) states. The experiments are simulated and the model prediction are compared with experimental data. Parameters for this simulation are collected from the literature [23] by Mašín, who compared his two hypoplastic models for clay using this set of data.

Predictions of the drained and undrained triaxial compression and extension tests, together with experimental results, are shown in Figure 9. It can be seen that the model predictions agree very well with the experimental results. Especially, excellent agreements are achieved in the stress - strain curves of the drained tests. On the other hand, this model underestimates the shear strength in undrained triaxial extension test for OCR = 12.

5.4. CID and CIU test, undisturbed Bangkok stiff clay

In the previous evaluation, the model is validated with different types of reconstituted clays. This model is able to capture most properties of reconstituted OC clay. To

Figure 8: Comparison between experimental and numerical drained constant mean pressure triaxial compression and extension tests on OC Fujinomori clay, data from Nakai and Hinoko [25]

broaden the validation of the proposed model, the final set of modeling will focus on an undisturbed clay, Bangkok stiff clay, from Bong Ngoo Hao site and Rangsit site. According to the original test results [13], although taken from different sites, these two soils exhibit similar overconsolidation behavior. In this study, selected triaxial compression test are simulated using the proposed model.

The reported peak friction angle and cohesion for samples from Bong Ngoo Hao site are $\phi_p = 26^\circ$ and $c = 30$ kPa, respectively. This friction angle is apparently higher than the critical friction angle. Considering the high cohesion value and taking into account that the failure envelope for stiff, OC clays is curvilinear, a critical friction angle $\phi_c = 31.5^\circ$ is adopted for the Bong Ngoo Hao stiff clay. Results from drained and undrained triaxial tests on these samples show that a slightly lower critical friction angle was obtained for samples from the Rangsit site. These results are adopted in our simulation. The parameters λ^* and κ^* are calibrated from oedometer tests presented in [35], and the other parameters are listed in Table 1.

Comparison between numerical and experimental results of the drained compression tests on undisturbed Bangkok clay from Bong Ngoo Hao site is presented in Figure

Figure 9: Comparison between experimental and numerical drained (a,b) and undrained (c,d) triaxial tests on remoulded Weald clay. Experimental data from Parry and Aust [29]

10. The adopted pre-consolidation pressure for the simulations is 700 kPa with various confining pressures of 35, 105, and 414 kPa. Inspection of the results suggests that the undisturbed clay samples show evident strain softening behavior. Particularly, for the sample under confining pressure 414 kPa, a sudden failure at an axial strain of 0.06 is observed. This may be most likely due to the natural structure of intact samples. The proposed model underestimates the peak strength for the samples with confining pressures of 35 and 105 kPa, while overpredicts the peak strength for the samples with confining pressure 414 kPa. However, an excellent agreement with volume change is observed in the simulations.

The model is further validated with experimental results obtained from undrained triaxial tests on samples taken from the Rangsit site. Seven different confining pressures, i.e., 14, 70, 140, 207, 276, 345, 414 kPa, with the same pre-consolidation pressure of 700 kPa are used in this simulation. Comparison between predictions and experimental results is shown in Figure 11. Note that the results for confining pressures of 140, 276, and 345 kPa are not shown in the $p/q - \varepsilon_1$ curves because these testing results showed significant deviation from the general experimental trend. The corresponding effective stress paths are repented in the $p - q$ curves with solid symbols. An excellent agreement with experimental results is achieved. This again suggests that the proposed model can

Figure 10: Comparison between experimental and numerical drained triaxial compression tests on undisturbed Bangkok stiff clay at Bong Ngoo Hao site. Experimental data from Hassan [13]

correctly predict the behaviors of OC clays.

6. Conclusions

In this paper, we present extend a basic hypoplastic constitutive model for OC clays. New density and stiffness factors are introduced to account for consolidation history. The model is capable of predicting a desirable ASBS. The shape of the ASBS is close to the elliptic shape of the yield surface of the MCC model on the wet side, while similar to the nonlinear Hvorslev surface on the dry side. This feature ensures that the model can properly predict the peak and critical strengths of OC clays. This model is validated by experiments on both reconstituted and undisturbed OC clays. The validation shows that the proposed model can capture the most common features of OC clays.

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Figure 11: Comparison between experimental and numerical undrained triaxial compression tests on undisturbed Bangkok stiff clay at Rangsit site. Experimental data from Hassan [13]

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Appendix

The concrete model formulation is:

$$\dot{\mathbf{T}} = m_R f_s \mathcal{L} : \left(\mathbf{D} - f_d Y \frac{\mathbf{m}}{\|\mathbf{m}\|} \|\mathbf{D}\| \right) \quad (.1)$$

The tensor $\mathcal{L}$ is:

$$\mathcal{L} = c_1 \mathcal{I} + c_2 \hat{\mathbf{T}} \otimes \mathbf{1} + c_3 \hat{\mathbf{T}} \otimes \hat{\mathbf{T}} \quad (.2)$$

where the normalized stress $\hat{\mathbf{T}} = \mathbf{T} / \text{tr} \mathbf{T}$. $\mathcal{I}$ and $\mathbf{1}$ are the fourth-order and the second-order unit tensor, respectively. The constants c_1 , c_2 , and c_3 are:

$$c_1 = \frac{2\sqrt{3}a(\gamma - 1)}{9v_i}, \quad c_2 = \frac{\sqrt{3}a\gamma - a^2 - 3}{3}, \quad c_3 = a^2 + 3 - 3c_1 \quad (.3)$$

The hypoplastic flow rule $\mathbf{m}$ is

$$\mathbf{m} = -\hat{\mathbf{T}}^* - \frac{F^2 - a^2 \|\hat{\mathbf{T}}^*\|^2}{F^2(1 + c_2) + a^2 \|\hat{\mathbf{T}}\|^2} \hat{\mathbf{T}} \quad (.4)$$

with the interpolation factor F given as

$$F = \left[\frac{3}{8} \|\hat{\mathbf{T}}^*\|^2 + \frac{1 - 1.5 \|\hat{\mathbf{T}}^*\|^2}{1 + \sqrt{1.5} \|\hat{\mathbf{T}}^*\| \cos 3\theta} \right]^{1/2} - \sqrt{\frac{3}{8}} \|\hat{\mathbf{T}}^*\| \quad (.5)$$

where the Lode angle θ is defined as:

$$\cos 3\theta = -\sqrt{6} \frac{\text{tr}(\hat{\mathbf{T}}^* \cdot \hat{\mathbf{T}}^* \cdot \hat{\mathbf{T}}^*)}{\|\hat{\mathbf{T}}^*\|^{3/2}} \quad (.6)$$

The limit stress condition Y is given as:

$$Y = \frac{(\gamma - 1)(I_1 I_2 / I_3 - 9) + 8 \tan^2 \phi_c}{8 \gamma \tan^2 \phi_c} \quad (.7)$$

where I_1 , I_2 , and I_3 are stress invariants expressed as:

$$I_1 = \text{tr} \mathbf{T}, \quad I_2 = [\mathbf{T} : \mathbf{T} - (I_1)^2]/2, \quad I_3 = \det \mathbf{T} \quad (.8)$$

The multipliers f_s , f_d , and m_R are:

$$f_s = \frac{\sqrt{3} \text{tr} \mathbf{T}}{2a} \left(\frac{1}{\lambda^*} - \frac{1}{\kappa^*} \right), \quad f_d = \frac{1}{R^\alpha}, \quad m_R = \exp \left(1 - \frac{1}{R} \right) \quad (.9)$$

with the stress ratio R being defined as:

$$R = \frac{p_e^+}{p_e^*} \quad (.10)$$

where p_e^+ is the pre-consolidation pressure in the MCC model, and p_e^* is the Hvorslev equivalent pressure.

$$p_e^+ = p \left[1 + \left(\frac{q}{Mp} \right)^2 \right], \quad p_e^* = \exp \left[\frac{N - \ln(1 + e)}{\lambda^*} \right] \quad (.11)$$

In the above equations, γ , a , and M are material-related constants:

$$\gamma = \frac{\lambda^* + \kappa^*}{\lambda^* - \kappa^*}, \quad a = \frac{\sqrt{3}(3 - \sin \phi_c)}{2\sqrt{2} \sin \phi_c}, \quad \text{and } M = \frac{3\sqrt{3}F}{a} \quad (.12)$$

Finally, there are six parameters for the proposed model: ϕ_c , λ^* , κ^* , N , v_i , and α .

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